1030 U2 random variable+binomial distribution

1. In the United States, the generation of people born between 1946 and 1964 are known as baby boomers, and the generation of people born between 1981 and 1996 are known as millennials. Currently, 18 percent of the population are baby boomers and 27 percent of the population are millennials. A random sample of 500 people will be selected. Let the random variable B represent the number of baby boomers in the sample, and let the random variable M represent the number of millennials in the sample. By how much will the mean of M exceed the mean of B ?

(A) 9

(B) 45 ✓

(C) 90

(D) 135

Answer B

Correct. The mean of B is $np = 500\,(0.18) = 90$. The mean of M is $np = 500\,(0.27) = 135$. The difference of the means is $135 - 90 = 45$.

2. According to a recent survey, 31 percent of the residents of a certain state who are age 25 years or older have a bachelor's degree. A random sample of 50 residents of the state, age 25 years or older, will be selected. Let the random variable B represent the number in the sample who have a bachelor's degree. What is the probability that B will equal 40 ?

(A) $\binom{50}{40}(0.31)^{40}(0.69)^{10}$ ✓

(B) $\binom{50}{40}(0.69)^{40}(0.31)^{10}$

(C) $\binom{40}{10}(0.31)^{40}(0.69)^{10}$

(D) $\binom{40}{10}(0.69)^{40}(0.31)^{10}$

Answer A

Correct. Define success as selecting a person with the degree and failure as selecting a person without the degree. Random variable B has a binomial distribution, with probability of success of 0.31 and probability of failure of 0.69. The probability of 40 out of 50 people having a degree is given by $\binom{50}{40}(0.31)^{40}(0.69)^{10}$.

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3. A blind taste test will be conducted with 9 volunteers to determine whether people can taste a difference between bottled water and tap water. Each participant will taste the water from two different glasses and then identify which glass he or she thinks contains the tap water. Assuming that people cannot taste a difference between bottled water and tap water, what is the probability that at least 8 of the 9 participants will correctly identify the tap water?

(A) 0.0020

(B) 0.0195 ✓

(C) 0.8889

(D) 0.9980

Answer B

4. In a certain region, 94 percent of the people have a certain characteristic in their blood. Suppose a group of 45 people from the region are selected at random. Let the random variable B represent the number of people in the sample without the characteristic. Random variable B follows a binomial distribution with a mean of 2.7 people. Which of the following is the best interpretation of the mean?

(A) For all groups of 45 people, the average number of people without the characteristic is 2.7. ✓

(B) Every group of 45 people will have 2.7 people without the characteristic.

(C) On average, 2.7 people are selected until finding someone with the characteristic.

(D) On average, 2.7 people are selected until finding someone without the characteristic.

Answer A

Correct. For the random variable B, the mean number of people without the characteristic is equal to 2.7 for all possible groups of 45 people.

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5. The following table shows the probability distribution for the number of books a student typically buys at the annual book fair held at an elementary school.

Number of Books	0	1	2	3	4	5	6	7
Probability	0.35	0.20	0.15	0.10	0.07	0.08	0.04	0.01

Let the random variable B represent the number of books a student buys at the next book fair. What is the expected value of B ?

(A) 0

(B) 1.00

(C) 1.79 ✓

(D) 3.50

Answer C

Correct. The expected value of B is the sum of the products of the number of books and their respective probabilities. In this case, $0(0.35) + 1(0.20) + 2(0.15) + 3(0.10) + 4(0.07) + 5(0.08) + 6(0.04) + 7(0.01) = 1.79$.

6. A company that ships crystal bowls claims that bowls arrive undamaged in 95 percent of the shipments. Let the random variable G represent the number of shipments with undamaged bowls in 25 randomly selected shipments. Random variable G follows a binomial distribution with a mean of 23.75 shipments and a standard deviation of approximately 1.09 shipments. Which of the following is the best interpretation of the mean?

(A) Every shipment of 25 bowls will have 23.75 undamaged bowls.

(B) On average, the company receives 23.75 shipments before receiving the first shipment with a damaged bowl.

(C) For all possible shipments of size 25, the average number of damaged shipments is equal to 23.75.

(D) For all possible shipments of size 25, the average number of undamaged shipments is equal to 23.75. ✓

Answer D

Correct. For the random variable G, the mean number of undamaged bowls is equal to 23.75 for all possible shipments of size 25.

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7. Let the random variable X represent the amount of money won or lost for a player who pays \$1 to play a certain carnival game. The following table shows the probability distribution of X.

Amount	$-\$1$	\$1	\$10
Probability	0.80	0.15	0.05

Which of the following statements is the best interpretation of the mean of X ?

(A) In the long run, a player will lose an average of \$0.15 per carnival game played. ✓

(B) In the long run, a player will gain an average of \$0.15 per carnival game played.

(C) In the long run, a player will lose an average of \$0.80 per carnival game played.

(D) In the long run, players will lose \$1 about 80 percent of the time.

Answer A

Correct. The expected value is $\Sigma x P(x) = -\$1(0.80) + \$1(0.15) + \$10(0.05) = -\$0.15$, indicating average loss. So the player can expect to lose an average of \$0.15 per carnival game when the game is played many times.

8. According to 2015 census data, 42.7 percent of Colorado residents were born in Colorado. If a sample of 250 Colorado residents is selected at random, what is the standard deviation of the number of residents in the sample who were born in Colorado?

(A) 6.75

(B) 7.82 ✓

(C) 10.33

(D) 11.97

Answer B

Correct. Let random variable C represent the number of people from a sample of 250 who were born in Colorado. The random variable has a binomial distribution. The standard deviation of a binomial random variable is $\sqrt{np(1-p)} = \sqrt{250(0.427)(0.573)} \approx 7.82$

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9.

Number of cookies sold	50	100	200
Probability	0.60	0.35	0.05

The probability distribution of the number of cookies sold by a store each day is shown in the table. The store earns a profit of $\$0.50$ per cookie sold.

What is the store's expected daily profit for the sales of the cookies?

(A) $\$37.50$ ✓

(B) $\$75.00$

(C) $\$116.67$

(D) $\$175.00$

Answer A

Correct. The expected number of cookies sold by the store each day is found by summing the products of the number of cookies sold and their probabilities, $50(0.60) + 100\,(0.35) + 200\,(0.05) = 75.00$. The expected daily profit is found by multiplying the constant price per cookie by the expected daily number of cookies sold, $0.50(75.00) = 37.50$.

10. A popular computer card game keeps track of the number of games played and the number of games won on that computer. The cards are shuffled before each game, so the outcome of the game is independent from one game to the next and is based on the skill of the player. Let X represent the number of games that have been won out of 100 games. Under which of the following situations would X be a binomial random variable?

(A) All games were played by the same player, whose skill improved over the course of the 100 games

(B) A group of 5 players of different skill levels were each allowed to play 20 games in a row.

(C) Two players of equal skill level each played one game a day for 50 days and their skill level did not change from day to day. ✓

(D) Two players of different skill levels competed by allowing one player to continue until a game was lost, then the other player to continue until a game was lost, and so on, until 100 games were played.

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11. A certain factory that manufactures office chairs has a quality control process to identify defective chairs. The binomial random variable D represents the number of chairs in a sample of chairs that are defective. The mean of D is 10 chairs and the standard deviation is 3 chairs. Based on the distribution of D, which of the following would be an accurate interpretation of the value 0.1 ?

(A) The total number of defective chairs made

(B) The relative frequency of the sample size to the population of chairs

(C) The probability of identifying a non-defective chair

(D) The probability of identifying a defective chair ✓

Answer D

Correct. The mean of 10 indicates $np = 10$, and the standard deviation of 3 indicates $\sqrt{np(1-p)} = \sqrt{10(1-p)} = 3$. Solving for p gives $p = 0.1$, which is the probability of success, that is, identifying a defective chair.

12. At a certain bakery, the price of each doughnut is \$1.50. Let the random variable D represent the number of doughnuts a typical customer purchases each day. The expected value and variance of the probability distribution of D are 2.6 doughnuts and 3.6 $(\text{doughnuts})^2$, respectively. Let the random variable P represent the price of the doughnuts that a typical customer purchases each day. Which of the following is the standard deviation, in dollars, of the probability distribution of P ?

(A) $1.5(3.6)$

(B) $1.5\sqrt{3.6}$ ✓

(C) $\sqrt{1.5(3.6)}$

(D) $1.5(2.6)$

Answer B

Correct. This is the price of each doughnut, \$1.50, multiplied by the standard deviation of the number of doughnuts $\sqrt{3.6}$.

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13. A magazine has 1,620,000 subscribers, of whom 640,000 are women and 980,000 are men. Thirty percent of the women read the advertisements in the magazine and 50 percent of the men read the advertisements in the magazine. A random sample of 100 subscribers is selected. What is the expected number of subscribers in the sample who read the advertisements?

(A) 40

(B) 42 ✓

(C) 50

(D) 80

14. Let random variable U represent the field goal percentage (percentage of shots made) for players in a basketball league. The following table shows the probability distribution of the random variable U.

Field Goal Percentage	Probability
0.3	0.10
0.4	0.45
0.5	0.30
0.6	0.10
0.7	0.05

Fatima claims that the distribution of U is uniform with a median of 0.4 field goal percentage. Is Fatima's claim supported by the table?

(A) Yes, the distribution is uniform with a median of 0.4 field goal percentage.

(B) No, the distribution is skewed to the right with a median of 0.4 field goal percentage. ✓

(C) No, the distribution is skewed to the right with a median of 0.5 field goal percentage.

(D) No, the distribution is skewed to the left with a median of 0.4 field goal percentage.

Answer B

Correct. Fatima's claim is not supported by the table. A relative frequency histogram of the data will display most of the weight over the bars associated with a field goal percentage of 0.3 and 0.4 with a tail from 0.5 to 0.7, indicating a right skew. The median is equal to 0.4 field goal percentage, because 0.4 is the middle value of the distribution.

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15. In a certain game, a fair die is rolled and a player gains 20 points if the die shows a "6." If the die does not show a "6," the player loses 3 points. If the die were to be rolled 100 times, what would be the expected total gain or loss for the player?

(A) A gain of about 583 points

(B) A gain of about 83 points ✓

(C) A loss of about 250 points

(D) A loss of about 300 points

16. Data were collected on the number of days per week that members visit a certain fitness center. The values varied from 0 to 7, and a distribution of relative frequencies for the values was created. Let the random variable X represent the number of days per week that a member visits. The mean of X is 3.12. Which of the following statements is the best interpretation of the mean?

(A) Each member visits the fitness center 3 or 4 days per week.

(B) The average number of days that each member visits the fitness center is 3.12 days per week.

(C) The long-run average resulting from repeated sampling of members of the fitness center will approach 3.12 days per week. ✓

(D) For a random sample of members selected from the population, the average number of visits for the sample will be 3.12 days per week.

Answer C

Correct. The mean of a random variable is the long-run average based on repeated sampling from the population.

17. In 2014, 85 percent of households in the United States had a computer. For a randomly selected sample of 200 households in 2014, let the random variable C represent the number of households in the sample that had a computer. What are the mean and standard deviation of C ?

(A) The mean is 144.5 households, and the standard deviation is 5.05 households.

(B) The mean is 144.5 households, and the standard deviation is 13.04 households.

(C) The mean is 170 households, and the standard deviation is 5.05 households. ✓

(D) The mean is 170 households, and the standard deviation is 0.36 household.

Answer C

Correct. Random variable C has a binomial distribution. The mean of the random variable is $np = 200\,(0.85)$, and the standard deviation of the random variable is $\sqrt{np(1-p)} = \sqrt{200(0.85)\,(0.15)}$.

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18. A recent study reported that 45 percent of adults in the United States now get all their news online. A random sample of 8 adults in the United States will be selected. What is the probability that 6 of the selected adults get all their news online?

(A) $\binom{6}{2}(0.45)^6(0.55)^2$

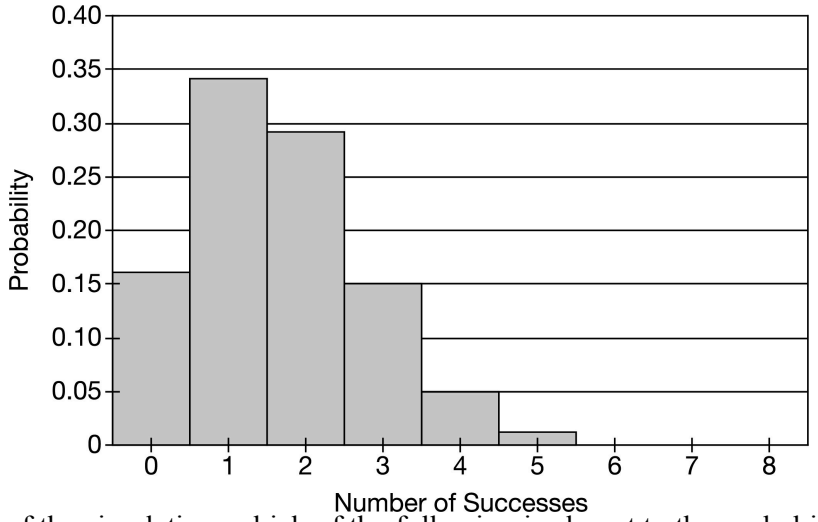
(B) $\binom{8}{6}(0.45)^2(0.55)^8$

(C) $\binom{8}{6}(0.45)^6(0.55)^2$ ✓

(D) $\binom{8}{6}(0.45)^8(0.55)^6$

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19. An experiment was conducted in which planks of wood painted red and green were shown to pigeons to investigate a pigeon's ability to select a certain color. Pigeons could accurately select the color of the plank of wood 20 percent of the time. A simulation was conducted in which a trial consisted of a pigeon being shown eight planks of wood and its number of successes being recorded. This process was repeated many times, and the results are shown in the histogram.



Based on the results of the simulation, which of the following is closest to the probability that there were at most three successes in a trial?

(A) 0.15

(B) 0.21

(C) 0.79

(D) 0.94 ✓

Answer D

Correct. The bars corresponding to 0, 1, 2, and 3 represent at most three successes. The sum of the heights of the bars corresponds to a probability of approximately 0.94.

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20. Voters in a certain large county will have an opportunity to vote for or against a new environmental rule that would incentivize the return of certain used batteries to a county-certified recycling facility. The results of a previous poll found that 41 percent of registered voters in that county plan to vote for the environmental rule. Juwan, an engineer working at the recycling facility, randomly selected 15 registered voters and asked whether they plan to vote for the rule.

Assuming that 41 percent represents the actual percent of registered voters in the county who plan to vote for the environmental rule, which of the following is closest to the probability that at most 2 of the registered voters Juwan selected plan to vote for the environmental rule?

(A) 0.0042

(B) 0.0185

(C) 0.0227 ✓

(D) 0.9773

Answer C

Correct. The number of registered voters Juwan selected who plan to vote for the environmental rule is a binomial random variable, X, with $n = 15$ and $p = 0.41$. The probability that at most 2 of the registered voters Juwan selected plan to vote for the environmental rule is

$$P\left(X \leq 2\right) = \sum_{x=0}^{2} \binom{15}{x} (0.41)^{x} (0.59)^{15-x} = 0.0004 + 0.0038 + 0.0185 = 0.0227.$$