1. A city has 48,000 registered voters. Suppose that 46 percent of them support a proposal to add protected bike lanes downtown. A local newspaper plans to select a random sample of registered voters and record the proportion of voters in the sample who support the proposal.

A. The newspaper selects a random sample of 150 registered voters. Let $\hat{p}$ be the proportion of voters in the sample who support the proposal.

i. What is the mean of the sampling distribution of $\hat{p}$?

ii. Calculate the standard deviation of the sampling distribution of $\hat{p}$. Verify that the condition needed to use this formula is met. Show your work.

B.

i. Is the sampling distribution of $\hat{p}$ approximately normal? Justify your answer.

ii. Determine the probability that a majority of the voters in the sample (more than 50 percent) support the proposal, even though only 46 percent of all registered voters do. Show your work.

C. The newspaper wants the standard deviation of the sampling distribution of $\hat{p}$ to be at most 0.02.

i. What is the smallest sample size that will achieve this? Show your work.

ii. A reporter says, “If we increase the sample size from 150 to the sample size in part C (i), the sample proportion will no longer be biased.” Explain why the reporter’s statement is incorrect, and describe what does change about the sampling distribution of $\hat{p}$ when the sample size increases.

Response space for Question 1

2. A high school wants to compare two ways of reminding students to submit homework online: a text-message reminder and an e-mail reminder. Two hundred students volunteer for an experiment. One hundred students are randomly assigned to receive text reminders, and the other 100 receive e-mail reminders. At the end of one week, the school records whether each student submitted every homework assignment on time. Suppose that the true proportion of students who would submit every assignment on time is 0.78 with text reminders and 0.70 with e-mail reminders. Let $\hat{p}_T$ and $\hat{p}_E$ be the proportions of students in the text group and the e-mail group, respectively, who submit every assignment on time.

A.

i. What is the mean of the sampling distribution of $\hat{p}_T - \hat{p}_E$?

ii. Calculate the standard deviation of the sampling distribution of $\hat{p}_T - \hat{p}_E$. Show your work.

B.

i. Is the sampling distribution of $\hat{p}_T - \hat{p}_E$ approximately normal? Justify your answer.

ii. Determine the probability that the e-mail group will have a higher proportion of on-time submissions than the text group in this experiment. Show your work.

C.

i. A student argues, “Because we subtract the two sample proportions, the standard deviation of $\hat{p}_T - \hat{p}_E$ must be smaller than the standard deviation of $\hat{p}_T$ and smaller than the standard deviation of $\hat{p}_E$.” Calculate the standard deviations of $\hat{p}_T$ and $\hat{p}_E$, and use them to explain why the student’s argument is incorrect.

ii. Suppose instead that only 25 students had been randomly assigned to each reminder method. Describe how the center, the variability, and the shape of the sampling distribution of $\hat{p}_T - \hat{p}_E$ would differ from your answers to parts A and B. Justify each answer.

Response space for Question 2

END OF QUESTIONS