1. A city has 48,000 registered voters. Suppose that 46 percent of them support a proposal to add protected bike lanes downtown. A local newspaper plans to select a random sample of registered voters and record the proportion of voters in the sample who support the proposal.

A. The newspaper selects a random sample of 150 registered voters. Let $\hat{p}$ be the proportion of voters in the sample who support the proposal.

i. What is the mean of the sampling distribution of $\hat{p}$?

ii. Calculate the standard deviation of the sampling distribution of $\hat{p}$. Verify that the condition needed to use this formula is met. Show your work.

Part	Model solution	Scoring
A	(i) $\mu_{\hat{p}} = p = 0.46$. (ii) $\sigma_{\hat{p}} = \sqrt{\frac{0.46(0.54)}{150}} \approx 0.0407$. The sample is random and 150 ≤ 0.10(48,000) = 4,800, so the 10% condition is met and the formula is appropriate.	**E** if the response: 1. gives the mean 0.46; 2. gives the SD ≈ 0.041 with work; 3. checks the 10% condition by comparing 150 with 0.10 × 48,000. **P** if exactly two of the three components are satisfied. **I** otherwise.

B.

i. Is the sampling distribution of $\hat{p}$ approximately normal? Justify your answer.

ii. Determine the probability that a majority of the voters in the sample (more than 50 percent) support the proposal, even though only 46 percent of all registered voters do. Show your work.

Part	Model solution	Scoring
B	(i) Yes. $np = 150(0.46) = 69 \geq 10$ and $n(1-p) = 150(0.54) = 81 \geq 10$, so the Large Counts condition is met. (ii) $z = \frac{0.50 - 0.46}{0.0407} \approx 0.98$ $P(\hat{p} > 0.50) = P(z > 0.98) \approx$ **0.163**. (normalcdf(lower = 0.50, upper = 1000, μ = 0.46, σ = 0.0407) ≈ 0.163)	**E** if the response: 1. states approximately normal and checks *both* Large Counts values; 2. identifies a normal model with mean 0.46, SD ≈ 0.041, boundary 0.50 and direction “>”; 3. gives the probability ≈ 0.16. **P** if exactly two of the three components are satisfied. **I** otherwise. Follow-through from an incorrect SD in part A is allowed.

C. The newspaper wants the standard deviation of the sampling distribution of $\hat{p}$ to be at most 0.02.

i. What is the smallest sample size that will achieve this? Show your work.

ii. A reporter says, "If we increase the sample size from 150 to the sample size in part C (i), the sample proportion will no longer be biased." Explain why the reporter's statement is incorrect, and describe what does change about the sampling distribution of $\hat{p}$ when the sample size increases.

Part	Model solution	Scoring
C	(i) $\sqrt{\frac{0.46(0.54)}{n}} \le 0.02 \Rightarrow n \ge \frac{0.2484}{0.02^2} = 621$, so **n = 621**. (621 ≤ 4,800, so the 10% condition still holds.) (ii) $\hat{p}$ is already unbiased at n = 150: the mean of its sampling distribution equals p = 0.46 for every sample size. A larger sample reduces the **variability**: the SD drops from about 0.041 to 0.020, so values of $\hat{p}$ cluster more closely around 0.46.	**E** if the response: 1. sets up $\sqrt{\frac{0.46(0.54)}{n}} \le 0.02$ (or equivalent) and gets n = 621; 2. states that $\hat{p}$ is unbiased (center = p) for both sample sizes; 3. states that the larger sample reduces the variability of $\hat{p}$. **P** if exactly two of the three components are satisfied. **I** otherwise. *Note:* n = 625 (using p = 0.5) satisfies component 1 only if it is explained as a conservative choice.

Scoring. Each part (A, B, C) is scored essentially correct (E), partially correct (P), or incorrect (I).

4	**Complete response:** three parts essentially correct
3	**Substantial response:** two parts essentially correct and one part partially correct
2	**Developing response:** two parts essentially correct and no parts partially correct; *or* one part essentially correct and one or two parts partially correct; *or* three parts partially correct
1	**Minimal response:** one part essentially correct and no parts partially correct; *or* no parts essentially correct and two parts partially correct
0	**No credit:** no parts essentially correct and fewer than two parts partially correct

2. A high school wants to compare two ways of reminding students to submit homework online: a text-message reminder and an e-mail reminder. Two hundred students volunteer for an experiment. One hundred students are randomly assigned to receive text reminders, and the other 100 receive e-mail reminders. At the end of one week, the school records whether each student submitted every homework assignment on time. Suppose that the true proportion of students who would submit every assignment on time is 0.78 with text reminders and 0.70 with e-mail reminders. Let $\hat{p}_T$ and $\hat{p}_E$ be the proportions of students in the text group and the e-mail group, respectively, who submit every assignment on time.

A.

i. What is the mean of the sampling distribution of $\hat{p}_T - \hat{p}_E$?

ii. Calculate the standard deviation of the sampling distribution of $\hat{p}_T - \hat{p}_E$. Show your work.

Part	Model solution	Scoring
A	(i) $\mu_{\hat{p}_T - \hat{p}_E} = 0.78 - 0.70 = 0.08$. (ii) $\sigma_{\hat{p}_T - \hat{p}_E} = \sqrt{\frac{0.78(0.22)}{100} + \frac{0.70(0.30)}{100}} \approx 0.0618$. Random assignment makes the results of the two groups independent, so the variances add.	**E** if the response: 1. gives the mean 0.08; 2. gives the SD ≈ 0.062 with both variance terms added. **P** if exactly one of the two components is satisfied. **I** otherwise. *Note:* the 10% condition is not required; the students were randomly assigned, not randomly sampled.

B.

i. Is the sampling distribution of $\hat{p}_T - \hat{p}_E$ approximately normal? Justify your answer.

ii. Determine the probability that the e-mail group will have a higher proportion of on-time submissions than the text group in this experiment. Show your work.

Part	Model solution	Scoring
B	(i) Yes. All four expected counts are at least 10: 100(0.78) = 78, 100(0.22) = 22, 100(0.70) = 70, 100(0.30) = 30. (ii) The e-mail group does better when $\hat{p}_T - \hat{p}_E < 0$. $z = \frac{0 - 0.08}{0.0618} \approx -1.29$ P($\hat{p}_T - \hat{p}_E < 0$) = P($z < -1.29$) ≈ **0.098**. (normalcdf(lower = −1000, upper = 0, μ = 0.08, σ = 0.0618) ≈ 0.098)	**E** if the response: 1. states approximately normal and checks all four counts; 2. translates the event as $\hat{p}_T - \hat{p}_E < 0$ and uses a normal model with mean 0.08 and SD ≈ 0.062; 3. gives the probability ≈ 0.098. **P** if exactly two of the three components are satisfied. **I** otherwise. Follow-through from part A is allowed.

C.

i. A student argues, "Because we subtract the two sample proportions, the standard deviation of $\hat{p}_T - \hat{p}_E$ must be smaller than the standard deviation of $\hat{p}_T$ and smaller than the standard deviation of $\hat{p}_E$." Calculate the standard deviations of $\hat{p}_T$ and $\hat{p}_E$, and use them to explain why the student's argument is incorrect.

ii. Suppose instead that only 25 students had been randomly assigned to each reminder method. Describe how the center, the variability, and the shape of the sampling distribution of $\hat{p}_T - \hat{p}_E$ would differ from your answers to parts A and B. Justify each answer.

Part	Model solution	Scoring
C	(i) $\sigma_{\hat{p}_T} = \sqrt{\frac{0.78(0.22)}{100}} \approx 0.0414$ $\sigma_{\hat{p}_E} = \sqrt{\frac{0.70(0.30)}{100}} \approx 0.0458$ The SD of the difference (≈ 0.062) is **larger** than both, because each sample proportion varies on its own and both sources of variability carry into the difference — the variances add. (ii) **Center:** unchanged, 0.08. **Variability:** larger: $\sigma_{\hat{p}_T - \hat{p}_E} = \sqrt{\frac{0.78(0.22)}{25} + \frac{0.70(0.30)}{25}} \approx 0.124,$ twice as large, because each group size is divided by 4. **Shape:** may not be approximately normal, because 25(0.22) = 5.5 and 25(0.30) = 7.5 are below 10.	**E** if the response: 1. gives both SDs (≈ 0.041, ≈ 0.046) and explains that the SD of the difference is larger because variability from both groups adds; 2. states the center is unchanged; 3. states the variability increases (≈ 0.124 or a factor of 2); 4. states the shape may not be normal, identifying a failing count. **P** if exactly two or three of the four components are satisfied. **I** otherwise.

Scoring. Each part (A, B, C) is scored essentially correct (E), partially correct (P), or incorrect (I).

4	**Complete response:** three parts essentially correct
3	**Substantial response:** two parts essentially correct and one part partially correct
2	**Developing response:** two parts essentially correct and no parts partially correct; *or* one part essentially correct and one or two parts partially correct; *or* three parts partially correct
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END OF SCORING GUIDELINES