Name: _____ Class / Date: _____

Every run: set the values → **clear** → draw **1,000 samples** → copy the numbers. Write every rule *from your own results*.

1 Simulator 1 — Sample proportion $\hat{p}$

A. Center

SIMULATE — Run each row and record the mean of the 1,000 values of $\hat{p}$.

Run	p	n	Simulated mean of $\hat{p}$	Run	p	n	Simulated mean of $\hat{p}$
1	0.30	20		3	0.70	50	
2	0.30	100		4	0.10	200	

a Compare Runs 1 and 2. Did increasing *n* move the center? What does decide where the center is?

RULE (center): $\mu_{\hat{p}}$ = _____ So $\hat{p}$ is *unbiased*, which means: _____

B. Spread

SIMULATE — Keep $p = 0.50$. Record the SD of $\hat{p}$ for each *n*.

n	25	100	400	When *n* is multiplied by 4, the SD is …
Simulated SD of $\hat{p}$				multiplied by _____

SIMULATE — Now keep $n = 100$ and change *p*. Fill in the last two rows by calculator.

p	0.10	0.30	0.50	0.70	0.90
Simulated SD of $\hat{p}$					
p(1 − p)					
$\sqrt{\frac{p(1-p)}{n}}$					

b Where is the SD largest? Where is it smallest? Which two values of *p* gave the same SD, and why?

RULE (spread), written as a fraction: $\sigma_{\hat{p}}$ = _____ To cut the SD in half, multiply *n* by _____.

C. The 10% Condition

Real samples are drawn **without replacement**. The SD rule above is only accurate when the sample is a small part of the population: **n ≤ 0.10 N** (N = population size). Always compare *n* with **0.10 N**, not with N.

Situation	n	N	0.10 N	Pass?
Poll of a school	60	500		
Survey of a city	400	80,000		

c If the 10% Condition fails, can you still calculate $\hat{p}$? Which number can you no longer trust?

D. Shape — how the population (p) shapes the histogram

SIMULATE — For each cell write the shape: **S** = symmetric, **R** = right-skewed, **L** = left-skewed; add **N** if it looks Normal (bell-shaped). Also fill in the two counts.

	n = 10	n = 40	n = 160
p = 0.50 (balanced)	shape: np = n(1−p) =	shape: np = n(1−p) =	shape: np = n(1−p) =
p = 0.10 (lopsided)	shape: np = n(1−p) =	shape: np = n(1−p) =	shape: np = n(1−p) =
p = 0.90 (lopsided)	shape: np = n(1−p) =	shape: np = n(1−p) =	shape: np = n(1−p) =

d When $p = 0.10$ and n is small, which way is the histogram skewed? Explain using the fact that $\hat{p}$ can never be below 0.

e Look at all the cells that looked Normal. What do their two counts have in common? Complete the rule.

RULE (Large Counts): $\hat{p}$ is approximately Normal when _____ ≥ _____ and _____ ≥ _____. Shape depends on _____ and _____ together.

2 Simulator 2 — Difference of proportions $\hat{p}_1 - \hat{p}_2$

A. Center

SIMULATE — Use $n_1 = n_2 = 50$. Record the mean of the 1,000 differences.

p_1	p_2	$p_1 - p_2$	Simulated mean of $\hat{p}_1 - \hat{p}_2$
0.60	0.35		
0.30	0.55		

RULE (center): $\mu_{\hat{p}_1 - \hat{p}_2}$ = _____ A negative center means: _____

B. Spread — the surprise

SIMULATE — Set $p_1 = p_2 = 0.50$ and $n_1 = n_2 = 100$.

① SD of each $\hat{p}$ alone (from Simulator 1): $\sigma_{\hat{p}_1} = \sigma_{\hat{p}_2}$ = _____

② Predict the SD of the difference — circle: 0 / 0.050 / 0.071 / 0.100 ③ Simulated SD of $\hat{p}_1 - \hat{p}_2$: _____

④ Test three guesses: $\sigma_1 - \sigma_2$ = _____ $\sigma_1 + \sigma_2$ = _____ $\sqrt{\sigma_1^2 + \sigma_2^2}$ = _____ ← which one matches ③?

f Why is the difference **more** variable than either sample on its own, even though we subtract? (Think: how many random things are moving?)

RULE (spread): variances _____. $\sigma_{\hat{p}_1 - \hat{p}_2}$ = [ ]

g Check your rule: $p_1 = 0.45$, $n_1 = 80$, $p_2 = 0.30$, $n_2 = 120$. Calculate the mean and SD of $\hat{p}_1 - \hat{p}_2$ (both fractions under one square root), then compare with the simulator.

C. Conditions and shape for a difference

SIMULATE — (i) $p_1 = 0.05$, $n_1 = 20$ with $p_2 = 0.50$, $n_2 = 200$. (ii) Same *p*'s, but $n_1 = 200$. Sketch both histograms of $\hat{p}_1 - \hat{p}_2$.

(i) $n_1 = 20$, $n_2 = 200$

(ii) $n_1 = n_2 = 200$

Four counts	n_1p_1	$n_1(1 - p_1)$	n_2p_2	$n_2(1 - p_2)$	All ≥ 10?
(i)					
(ii)					

h In (i), which sample spoiled the Normal shape? How many counts must you check for a difference, and what does one failing count do?

CONDITIONS for $\hat{p}_1 - \hat{p}_2$: 10% Condition for _____ sample(s); the two samples must be _____; Large Counts: all _____ counts ≥ 10.

3 Simulator 3 — Sample mean $\bar{x}$

Now each value is a number (height, score…). The population has mean μ, SD σ and a **shape** you choose in the simulator.

A. Center and spread

SIMULATE — Choose the **right-skewed** population. Copy the population values: μ = _____ σ = _____ . Then run each *n*.

n	Simulated mean of $\bar{x}$	Simulated SD of $\bar{x}$	$\frac{\sigma}{\sqrt{n}}$ (calculator)	Close to the simulated SD?
4				
16				
64				

a Did *n* change the center? Each time *n* was multiplied by 4, what happened to the SD? Is that the same pattern as for $\hat{p}$?

RULES: $\mu_{\bar{x}}$ = _____ $\sigma_{\bar{x}}$ = _____ (use only if the _____ Condition holds)

b Check: μ = 72, σ = 12, n = 36 from a population of N = 300. Find $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$. Is the SD formula allowed here? Show 0.10 N.

B. Shape — how the population shape affects the histogram of $\bar{x}$

SIMULATE — For each population shape run $n = 2, 10, 40$. Write **P** = still looks like the population, **B** = in between, **N** = Normal-looking.

Population shape	n = 2	n = 10	n = 40
Normal			
Uniform			
Right-skewed			
Bimodal			

c Which population gave a Normal-looking $\bar{x}$ even at $n = 2$? Which needed the largest n? Did the population shape change the *center* or the *SD* of $\bar{x}$, or only its *shape*?

RULE (shape): Normal population → $\bar{x}$ is Normal for _____ n. Other shapes → approx. Normal when $n \geq$ _____ (the _____ _____ Theorem). For $\hat{p}$, "population shape" means the value of _____.

4 Simulator 4 — Difference of means $\bar{x}_1 - \bar{x}_2$

SIMULATE — Population 1: Normal, $\mu_1 = 50$, $\sigma_1 = 10$, $n_1 = 40$. Population 2: Normal, $\mu_2 = 42$, $\sigma_2 = 8$, $n_2 = 40$. Predict first, then simulate.

	My prediction (show the calculation)	Simulator
Mean of $\bar{x}_1 - \bar{x}_2$		
SD of $\bar{x}_1 - \bar{x}_2$		

RULES: $\mu_{\bar{x}_1 - \bar{x}_2}$ = _____ $\sigma_{\bar{x}_1 - \bar{x}_2}$ = _____ means _____, variances _____

SIMULATE — Make both populations **right-skewed**. Run (i) $n_1 = 5$, $n_2 = 50$, then (ii) $n_1 = n_2 = 50$.

d Describe the shape of $\bar{x}_1 - \bar{x}_2$ in (i) and (ii). Which sample size controlled the shape in (i)? Which condition explains the change?

Summary — fill in from your own results

Statistic	Mean	SD (if 10% Condition holds)	Approximately Normal when …
$\hat{p}$			
$\hat{p}_1 - \hat{p}_2$			
$\bar{x}$			
$\bar{x}_1 - \bar{x}_2$			