1. A regional blood donation center has 50,000 registered donors. Seven percent of the donors in the center's region have type O-negative blood, the blood type that can be given to patients of any blood type.

A. Twenty donors are selected at random from the registered donors.

i. Define the random variable of interest for the number of donors in the sample who have type O-negative blood, and state how the random variable is distributed.

ii. Determine the probability that at least 3 of the 20 donors have type O-negative blood. Show your work.

B. Each month the center tests a random sample of 400 registered donors. Let $\hat{p}$ be the proportion of the sampled donors who have type O-negative blood.

i. Describe the sampling distribution of $\hat{p}$. Justify your answer.

ii. The center issues a shortage alert if fewer than 5 percent of the sampled donors in a month have type O-negative blood. Determine the probability that the center issues a shortage alert in a particular month. Show your work.

C. In a neighboring region, 10 percent of donors have type O-negative blood. That region's center tests a random sample of 300 of its donors, independently of the sample of 400 in part B. Let $\hat{p}_A$ be the sample proportion of O-negative donors in the sample of 400 from the first region, and let $\hat{p}_B$ be the sample proportion in the sample of 300 from the neighboring region.

i. Calculate the mean and the standard deviation of the sampling distribution of $\hat{p}_B - \hat{p}_A$. Show your work.

ii. Determine the probability that the sample from the first region has a proportion of O-negative donors at least as large as the sample from the neighboring region. Show your work.

Response space for Question 1

2. A bakery bakes bread in two ovens and produces thousands of loaves each day. The weights of loaves from Oven 1 are approximately normally distributed with mean 500 grams and standard deviation 12 grams. The weights of loaves from Oven 2 have a distribution that is skewed to the right with mean 495 grams and standard deviation 15 grams.

A. Consider the loaves from Oven 1.

i. Determine the probability that a randomly selected loaf from Oven 1 weighs less than 480 grams. Show your work.

ii. The bakery labels the heaviest 5 percent of loaves from Oven 1 as "extra large." What is the minimum weight of a loaf labeled "extra large"? Show your work.

B. An inspector selects a random sample of 9 loaves from Oven 1 and calculates the sample mean weight $\bar{x}$.

i. Describe the sampling distribution of $\bar{x}$. Justify your answer.

ii. Determine the probability that the mean weight of the 9 loaves is less than 492 grams. Explain why this probability is smaller than the probability that a single loaf weighs less than 492 grams.

C. The inspector also selects a random sample of 36 loaves from Oven 2, independently of the sample of 9 loaves from Oven 1. Let $\bar{x}_1$ be the mean weight of the 9 loaves from Oven 1 and $\bar{x}_2$ the mean weight of the 36 loaves from Oven 2.

i. Describe the sampling distribution of $\bar{x}_1 - \bar{x}_2$. Justify your answer.

ii. Determine the probability that the sample of loaves from Oven 2 has a larger mean weight than the sample of loaves from Oven 1. Show your work.

Response space for Question 2

END OF QUESTIONS