1. A regional blood donation center has 50,000 registered donors. Seven percent of the donors in the center's region have type O-negative blood, the blood type that can be given to patients of any blood type.

A. Twenty donors are selected at random from the registered donors.

i. Define the random variable of interest for the number of donors in the sample who have type O-negative blood, and state how the random variable is distributed.

ii. Determine the probability that at least 3 of the 20 donors have type O-negative blood. Show your work.

Part	Model solution	Scoring
A	(i) Let X = the number of the 20 randomly selected donors who have type O-negative blood. X has a binomial distribution with $n = 20$ and $p = 0.07$. (Each donor is O-negative or not; n is fixed at 20; p = 0.07 for each donor; donors are selected at random and $20 \le 0.10(50{,}000)$, so the trials are approximately independent.) (ii) $P(X \ge 3) = 1 - P(X \le 2) = 1 - 0.8390 \approx$ **0.161**. (1 − binomcdf(n = 20, p = 0.07, x = 2))	**E** if the response: 1. defines X in context (number of O-negative donors out of 20); 2. states binomial with n = 20 and p = 0.07; 3. gives $P(X \ge 3) \approx 0.161$ with the complement or the binomial setup shown. **P** if exactly two of the three components are satisfied. **I** otherwise. *Note:* $1 - P(X \le 3) \approx 0.045$ is a boundary error and does not satisfy component 3.

B. Each month the center tests a random sample of 400 registered donors. Let $\hat{p}$ be the proportion of the sampled donors who have type O-negative blood.

i. Describe the sampling distribution of $\hat{p}$. Justify your answer.

ii. The center issues a shortage alert if fewer than 5 percent of the sampled donors in a month have type O-negative blood. Determine the probability that the center issues a shortage alert in a particular month. Show your work.

Part	Model solution	Scoring
B	(i) **Center:** $\mu_{\hat{p}} = 0.07$. **Variability:** $\sigma_{\hat{p}} = \sqrt{\frac{0.07(0.93)}{400}} \approx 0.0128$; the formula is appropriate because $400 \le 0.10(50{,}000) = 5{,}000$. **Shape:** approximately normal, because $np = 400(0.07) = 28 \ge 10$ and $n(1-p) = 400(0.93) = 372 \ge 10$. (ii) $z = \frac{0.05 - 0.07}{0.0128} \approx -1.57$ $P(\hat{p} < 0.05) = P(z < -1.57) \approx$ **0.058**. (normalcdf(lower = −1000, upper = 0.05, μ = 0.07, σ = 0.0128))	**E** if the response: 1. gives center 0.07 and SD ≈ 0.0128, with the 10% condition checked; 2. states approximately normal, checking both Large Counts values; 3. uses the normal model with boundary 0.05 and direction "<", and gives ≈ 0.058. **P** if exactly two of the three components are satisfied. **I** otherwise.

C. In a neighboring region, 10 percent of donors have type O-negative blood. That region's center tests a random sample of 300 of its donors, independently of the sample of 400 in part B. Let $\hat{p}_A$ be the sample proportion of O-negative donors in the sample of 400 from the first region, and let $\hat{p}_B$ be the sample proportion in the sample of 300 from the neighboring region.

i. Calculate the mean and the standard deviation of the sampling distribution of $\hat{p}_B - \hat{p}_A$. Show your work.

ii. Determine the probability that the sample from the first region has a proportion of O-negative donors at least as large as the sample from the neighboring region. Show your work.

Part	Model solution	Scoring
C	(i) Mean: 0.10 − 0.07 = **0.03**. $\sigma_{\hat{p}_B - \hat{p}_A} = \sqrt{\frac{0.10(0.90)}{300} + \frac{0.07(0.93)}{400}} \approx 0.0215$ (variances add because the samples are independent). (ii) The event is $\hat{p}_B - \hat{p}_A \le 0$. The four counts 28, 372, 30 and 270 are all at least 10, so the sampling distribution is approximately normal. $z = \frac{0 - 0.03}{0.0215} \approx -1.39$; $P(\hat{p}_B - \hat{p}_A \le 0) \approx$ **0.082**. (normalcdf(lower = −1000, upper = 0, μ = 0.03, σ = 0.0215))	**E** if the response: 1. gives the mean 0.03; 2. gives the SD ≈ 0.0215 with both variance terms added; 3. translates the event as $\hat{p}_B - \hat{p}_A \le 0$ and gives ≈ 0.082, with a normal model justified by the four counts. **P** if exactly two of the three components are satisfied. **I** otherwise. A response that uses $\hat{p}_A - \hat{p}_B \ge 0$ with mean −0.03 is equivalent.

Scoring. Each part (A, B, C) is scored essentially correct (E), partially correct (P), or incorrect (I).

4	**Complete response:** three parts essentially correct
3	**Substantial response:** two parts essentially correct and one part partially correct
2	**Developing response:** two parts essentially correct and no parts partially correct; *or* one part essentially correct and one or two parts partially correct; *or* three parts partially correct
1	**Minimal response:** one part essentially correct and no parts partially correct; *or* no parts essentially correct and two parts partially correct
0	**No credit:** no parts essentially correct and fewer than two parts partially correct

2. A bakery bakes bread in two ovens and produces thousands of loaves each day. The weights of loaves from Oven 1 are approximately normally distributed with mean 500 grams and standard deviation 12 grams. The weights of loaves from Oven 2 have a distribution that is skewed to the right with mean 495 grams and standard deviation 15 grams.

A. Consider the loaves from Oven 1.

i. Determine the probability that a randomly selected loaf from Oven 1 weighs less than 480 grams. Show your work.

ii. The bakery labels the heaviest 5 percent of loaves from Oven 1 as "extra large." What is the minimum weight of a loaf labeled "extra large"? Show your work.

Part	Model solution	Scoring
A	(i) $z = \frac{480 - 500}{12} \approx -1.67$; P($X$ < 480) ≈ **0.048**. (normalcdf(lower = 0, upper = 480, μ = 500, σ = 12)) (ii) The 95th percentile: x = 500 + 1.645(12) ≈ **519.7 grams**. (invNorm(area = 0.95, μ = 500, σ = 12))	**E** if the response: 1. identifies the normal distribution with mean 500 and SD 12 for one loaf, with boundary 480 and direction "<"; 2. gives ≈ 0.048; 3. finds the 95th percentile ≈ 519.7 g, using z ≈ 1.645 or invNorm with area 0.95. **P** if exactly two of the three components are satisfied. **I** otherwise. *Note:* using the 5th percentile (≈ 480.3 g) does not satisfy component 3.

B. An inspector selects a random sample of 9 loaves from Oven 1 and calculates the sample mean weight $\bar{x}$.

i. Describe the sampling distribution of $\bar{x}$. Justify your answer.

ii. Determine the probability that the mean weight of the 9 loaves is less than 492 grams. Explain why this probability is smaller than the probability that a single loaf weighs less than 492 grams.

Part	Model solution	Scoring
B	(i) **Center:** $\mu_{\bar{x}}$ = 500 g. **Variability:** $\sigma_{\bar{x}} = \frac{12}{\sqrt{9}} = 4$ g (9 is less than 10% of the loaves produced). **Shape:** exactly normal, because the population of Oven 1 weights is normal — even though n = 9 is small. (ii) $z = \frac{492 - 500}{4} = -2$; P($\bar{x}$ < 492) ≈ **0.023**. For a single loaf, z = (492 − 500)/12 ≈ −0.67 and the probability is about 0.25. The mean of 9 loaves varies much less than an individual loaf (SD 4 g vs 12 g), so a mean as low as 492 is less likely.	**E** if the response: 1. gives center 500 and SD 4; 2. states normal because the population is normal (not because of the CLT, since n = 9 < 30); 3. gives ≈ 0.023 with the normal model for $\bar{x}$; 4. explains the comparison using the smaller variability of $\bar{x}$. **P** if exactly two or three of the four components are satisfied. **I** otherwise.

C. The inspector also selects a random sample of 36 loaves from Oven 2, independently of the sample of 9 loaves from Oven 1. Let $\bar{x}_1$ be the mean weight of the 9 loaves from Oven 1 and $\bar{x}_2$ the mean weight of the 36 loaves from Oven 2.

i. Describe the sampling distribution of $\bar{x}_1 - \bar{x}_2$. Justify your answer.

ii. Determine the probability that the sample of loaves from Oven 2 has a larger mean weight than the sample of loaves from Oven 1. Show your work.

Part	Model solution	Scoring
C	(i) **Center:** 500 − 495 = **5 g**. **Variability:** $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{12^2}{9} + \frac{15^2}{36}} = \sqrt{16 + 6.25} \approx 4.72$ g (independent samples, variances add). **Shape:** approximately normal: $\bar{x}_1$ is normal because Oven 1 weights are normal, and $\bar{x}_2$ is approximately normal by the central limit theorem because n = 36 ≥ 30, even though Oven 2 weights are skewed. (ii) The event is $\bar{x}_1 - \bar{x}_2 < 0$. $z = \frac{0 - 5}{4.72} \approx -1.06$; $P(\bar{x}_1 - \bar{x}_2 < 0) \approx$ **0.145**. (normalcdf(lower = −1000, upper = 0, μ = 5, σ = 4.72))	**E** if the response: 1. gives the mean 5 and SD ≈ 4.72 with both variance terms added; 2. justifies approximate normality for *both* sample means (population normal for Oven 1; CLT with n = 36 for Oven 2); 3. translates the event as $\bar{x}_1 - \bar{x}_2 < 0$ and gives ≈ 0.145. **P** if exactly two of the three components are satisfied. **I** otherwise.

Scoring. Each part (A, B, C) is scored essentially correct (E), partially correct (P), or incorrect (I).

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END OF SCORING GUIDELINES