1. An online learning platform has 80,000 registered students. Fifteen percent of the registered students use the platform's live-tutoring feature in a typical week.

A. Twenty-five registered students are selected at random.

i. Define the random variable of interest for the number of students in the sample who use the live-tutoring feature in a typical week, and state how the random variable is distributed.

ii. Determine the probability that at most 2 of the 25 students use the live-tutoring feature. Show your work.

B. Each semester the platform surveys a random sample of 300 registered students. Let $\hat{p}$ be the proportion of the sampled students who use the live-tutoring feature in a typical week.

i. Describe the sampling distribution of $\hat{p}$. Justify your answer.

ii. The platform reports "strong tutoring engagement" if more than 18 percent of the sampled students use the live-tutoring feature. Determine the probability that the platform reports strong tutoring engagement for a particular semester. Show your work.

C. On a second learning platform, 20 percent of registered students use live tutoring in a typical week. That platform surveys a random sample of 250 of its registered students, independently of the sample of 300 in part B. Let $\hat{p}_A$ be the sample proportion of tutoring users in the sample of 300 from the first platform, and let $\hat{p}_B$ be the sample proportion in the sample of 250 from the second platform.

i. Calculate the mean and the standard deviation of the sampling distribution of $\hat{p}_B - \hat{p}_A$. Show your work.

ii. Determine the probability that the sample from the first platform has a proportion of tutoring users at least as large as the sample from the second platform. Show your work.

Response space for Question 1

2. A company makes rechargeable batteries for wireless earbuds in two factories, and each factory produces thousands of batteries each day. The battery life (time until the charge runs out) of batteries from Factory 1 is approximately normally distributed with mean 20 hours and standard deviation 2.5 hours. The battery life of batteries from Factory 2 has a distribution that is skewed to the left with mean 19 hours and standard deviation 3 hours.

A. Consider the batteries from Factory 1.

i. Determine the probability that a randomly selected battery from Factory 1 lasts more than 24 hours. Show your work.

ii. The company will replace for free any battery whose life is in the shortest 2 percent of all Factory 1 batteries. What is the maximum battery life that qualifies for a free replacement? Show your work.

B. A quality inspector selects a random sample of 4 batteries from Factory 1 and calculates the sample mean battery life $\bar{x}$.

i. Describe the sampling distribution of $\bar{x}$. Justify your answer.

ii. Determine the probability that the mean battery life of the 4 batteries is more than 22.5 hours. Explain why this probability is smaller than the probability that a single battery lasts more than 22.5 hours.

C. The inspector also selects a random sample of 40 batteries from Factory 2, independently of the sample of 4 batteries from Factory 1. Let $\bar{x}_1$ be the mean battery life of the 4 batteries from Factory 1 and $\bar{x}_2$ the mean battery life of the 40 batteries from Factory 2.

i. Describe the sampling distribution of $\bar{x}_1 - \bar{x}_2$. Justify your answer.

ii. Determine the probability that the sample of batteries from Factory 2 has a longer mean battery life than the sample from Factory 1. Show your work.

Response space for Question 2

END OF QUESTIONS